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**МАТЕМАТИЧЕСКОЕ И ПРОГРАММНОЕ ОБЕСПЕЧЕНИЕ ВЫЧИСЛИТЕЛЬНЫХ СИСТЕМ,  
КОМПЛЕКСОВ И КОМПЬЮТЕРНЫХ СЕТЕЙ / MATHEMATICAL SOFTWARE FOR COMPUTERS,  
COMPLEXES AND COMPUTER NETWORKS**

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**DEVELOPMENT OF CONTROL SCHEMES FOR BASIC CRYPTOGRAPHIC TRANSFORMATION  
OPERATIONS PERFORMED BY A PROGRAMMABLE LOGIC INTEGRATED CIRCUIT**

Research article

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**Abstract**

This article discusses the procedure for developing control schemes for the basic operations of cryptographic transformation of information performed by a programmable logic integrated circuit that is part of the control system of an autonomous technical means. The purpose of the research is to analyze the basic logical operations used by various algorithms for cryptographic transformation of information, as well as to develop a mathematical apparatus for monitoring the correctness of these operations. The control of the execution of logical operations is carried out inside a programmable logic integrated circuit in parallel with the main algorithm of cryptographic transformation of information. In the future, the developed control schemes can be used in the development of an algorithm for technical diagnostics of programmable logic integrated circuits with localization of the location and cause of the malfunction.

**Keywords:** technical control, programmable logic integrated circuit, modular arithmetic, cryptographic transformation of information, technical diagnostics, autonomous technical tool.

**РАЗРАБОТКА СХЕМ УПРАВЛЕНИЯ БАЗОВЫМИ ОПЕРАЦИЯМИ КРИПТОГРАФИЧЕСКОГО  
ПРЕОБРАЗОВАНИЯ, ВЫПОЛНЯЕМЫМИ ПРОГРАММИРУЕМОЙ ЛОГИЧЕСКОЙ ИНТЕГРАЛЬНОЙ  
СХЕМОЙ**

Научная статья

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**Аннотация**

В данной статье рассматривается процедура разработки схем управления основными операциями криптографического преобразования информации, выполняемыми программируемой логической интегральной схемой, входящей в состав системы управления автономным техническим объектом. Целью исследования является анализ основных логических операций, используемых различными алгоритмами для криптографического преобразования информации, а также разработка математического аппарата для контроля корректности выполнения этих операций. Управление выполнением логических операций осуществляется внутри программируемой логической интегральной схемы параллельно с основным алгоритмом криптографического преобразования информации. В дальнейшем разработанные схемы управления могут быть использованы при разработке алгоритма технической диагностики программируемых логических интегральных схем с локализацией места и причины неисправности.

**Ключевые слова:** технический контроль, программируемая логическая интегральная схема, модульная арифметика, криптографическое преобразование информации, техническая диагностика, автономное техническое средство.

**Introduction**

Cryptographic transformation of information (hereinafter referred to as KPI) is the process of changing information that depends on the parameter being changed, and has the property that it is impossible to restore the original information from the transformed one, without knowing the current key, with a labor intensity less than the specified one.

Currently, KPI is implemented by software, hardware, and software-to-hardware tools. At the same time, in the case of even a minimal error of one bit, the information recovery process becomes impossible.

Especially relevant is the problem of error occurrence during KPI on autonomous technical means (hereinafter referred to as PBX) that operate at a long distance from the technical operator and transmit cryptographically transformed information to the processing object via radio channels.

In this regard, there is a need for autonomous monitoring of the implementation of KPI on the PBX in order to determine the error in advance and, if possible, eliminate the malfunction of the device that performs KPI.

The main computing device that performs KPI is currently a programmable logic integrated circuit. This computing tool is the most versatile for implementing various KPI algorithms.

The most cryptographic algorithms are based on four basic logical operations:

- modulo 2 addition operation.
- modulo  $2^{16}$  addition operation;
- cyclic shift operation.
- substitution operation.

Control of these logical operations will allow you to most fully determine the technical condition of the FPGA performed by the KPI on the PBX.

Consider numerical control of arithmetic operations modulo. The construction of control schemes is based on two theorems [1].

Theorem 1. The sum of numbers is comparable modulo  $q$  with the sum of the residuals  $r$  of the same numbers, i.e. [2]

$$\sum_{i=1}^n A_i = \sum_{i=1}^n r_{ai} \text{ mod } q. \tag{1}$$

Theorem 2. The product of numbers is comparable modulo  $q$  with the product of the residuals  $r$  of the same numbers, i.e. [2]

$$\prod_{i=1}^n A_i = \prod_{i=1}^n r_{ai} \text{ mod } q. \tag{2}$$

Let us consider control schemes for the following transformations: summation modulo 2, summation modulo  $2^{16}$ , substitution and cyclic shift operation [3], [4], [5].

**Results and Discussion**

Development of an algorithm for controlling the modulo summation operation. The control scheme for adding numbers modulo 2 is shown in Fig. 1.

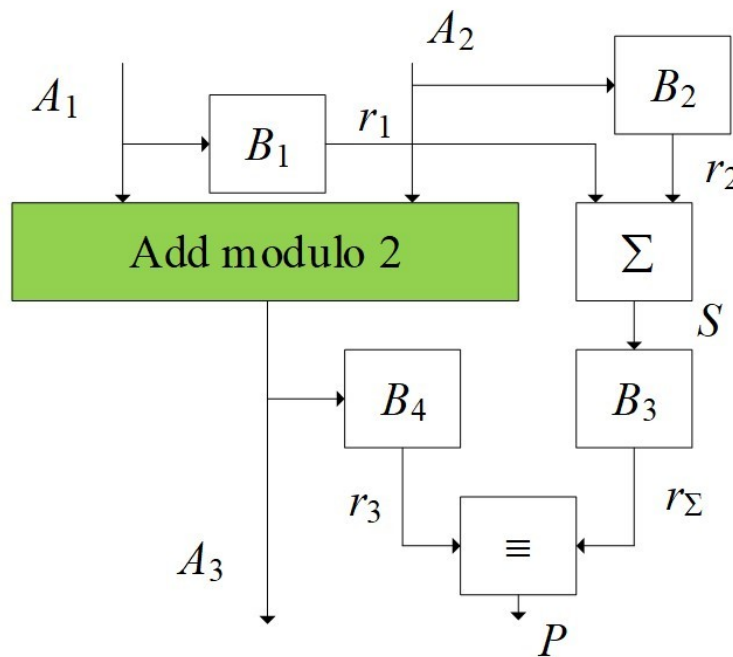


Figure 1 - Control of the value of binary numbers modulo 2  
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In general, the operation of this scheme can be described in the following way.

The result of summing two numbers  $A_1$  and  $A_2$  is the number  $A_3$ . In blocks  $B_1$  and  $B_2$ , the residuals  $r_1$  and  $r_2$  numbers  $A_1$  and  $A_2$  before conversion, respectively. Then, in block  $S$ , the resulting residuals are summed up. Since the sum of the residuals can be greater than the modulus, then at the output of the adder, it is necessary to perform the operation of finding the remainder again using the conversion in block  $B_3$ . As a result, the remainder  $r_{A_3}$  obtained from the number  $A_3$  is compared with the sum of the remainder of the numbers  $A_1$  and  $A_2 - r_{\Sigma}$  in the comparison module ( $\equiv$ ) with the formation of the "norm" sign ( $P$ ) in the form of a logical unit and zero – otherwise. It should be noted that the  $P$ -value is an additional feature (parameter) of the subsystem functioning and diagnostics model, i.e.  $P \in \Omega_{\zeta}$ .

The functioning of the presented control scheme can be divided into several stages:

1. At the initial stage, the numbers involved in the addition operation modulo 2 are obtained. After that, the remainder of the terms  $r_1, r_2$  and their sum ( $S = r_1 + r_2$ ) are found.

2. In the next step, we determine the remainder  $r_3$  of the sum of the residuals  $S$  and the remainder  $r_4$  of the sum of the terms  $A_1$  and  $A_2$ .

3. At the last stage, a report is generated about whether the summation is correct (output  $P_1$ ) or not (output  $P_2$ ). From a formal point of view, these stages can be represented as a set of the following maps [6]:

$$B_1 : A_1 \rightarrow r_1, \tag{3}$$

where  $B_1$  is an operator that characterizes getting the remainder  $r_1$  from a number  $A_1$ .

$$B_2 : A_2 \rightarrow r_2, \tag{4}$$

where  $B_2$  is an operator that characterizes getting the remainder  $r_2$  from a number  $A_2$ .

$$S : \sum_{i=1}^2 r_i \rightarrow s, s \in S \tag{5}$$

where  $S$  is the operator for finding the sum of balances  $r_1$  and  $r_2$ .

$$B_3 : S \rightarrow r_3, \tag{6}$$

where  $B_3$  is an operator that characterizes obtaining the remainder  $r_3$  from the sum of the remainder  $r_1$  and  $r_2$ .

$$P : r_3 \vee r_4 \rightarrow p, p \in \{1, 0\}, \tag{7}$$

where  $P$  is an operator that characterizes obtaining the solution  $p$ .

When controlling addition modulo  $2^{16}$ , the scheme is simplified, taking into account the fact that the remainder of two numbers is calculated in the transformation itself (Fig. 2).

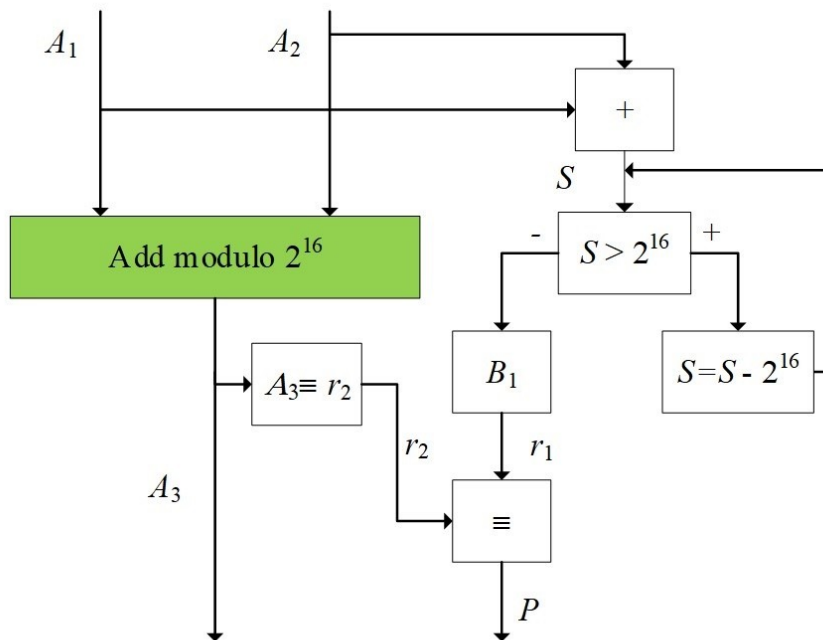


Figure 2 - Control of the position of binary numbers modulo  $2^{16}$   
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The functioning of the presented control scheme can be divided into several stages:

1. At the initial stage, the addition of numbers is performed. After that, the sum is compared with the number  $2^{16}$ . If the value is exceeded, the difference is calculated  $S > 2^{16} \Rightarrow S = S - 2^{16}$ .

2. In the next step, we define the remainder  $r_1$  of the sum of the numbers  $S$  and the remainder  $r_2$  as a number  $A_3$  ( $A_3 \equiv r_2$ ).

3. At the last stage, a report is generated about whether the summation is correct (output  $P_1$ ) or not (output  $P_2$ ).

Basic analytical relations [7]:

$$B_1 : S \rightarrow r_1, \tag{8}$$

where  $B_1$  is an operator that characterizes getting the remainder  $r_1$  from a number  $S$ .

$$P : r_1 \vee r_2 \rightarrow p, p \in \{1, 0\}, \tag{9}$$

where  $P$  is an operator that characterizes obtaining the solution  $p$ .

The control scheme of the shift operation is shown in Fig. 3.

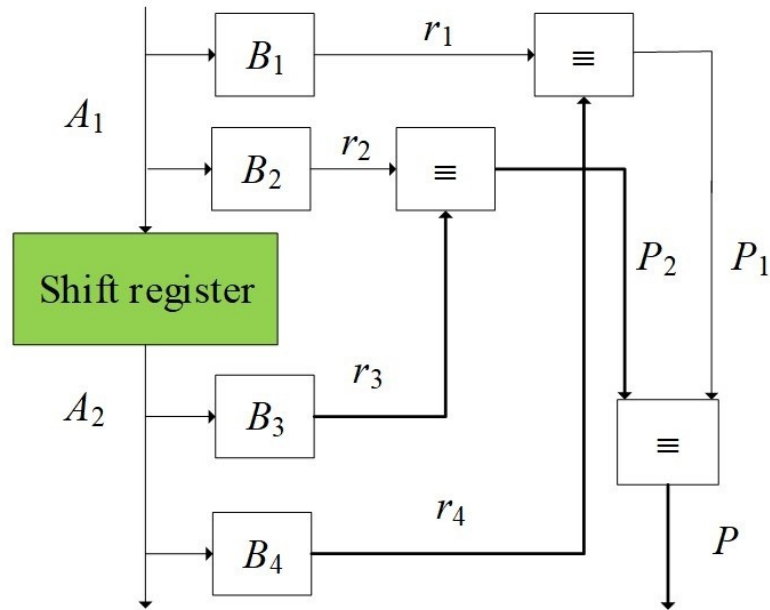


Figure 3 - Control scheme for the shift operation  
 DOI: <https://doi.org/10.60797/IRJ.2024.150.121.3>

The functioning of the presented control scheme can be divided into several stages:

1. At the initial stage, the number is represented  $A_i$  as two numbers relative to the number of clock cycles of the shift towards the highest digit. The residuals of the indicated numbers are calculated. Similarly, the component numbers are determined  $A_2$  and the residuals are calculated.

2. In the next step, the remainder of the numbers before and after the shift is compared.

3. At the last stage, a report is generated about whether or not the number shift ( $P$ ) is correct.

Basic analytical relations [8], [9]:

$$\begin{aligned} B_1 : A_1 &\rightarrow r_1 \\ B_2 : A_1 &\rightarrow r_2 \end{aligned} \quad (10)$$

where  $B_1, B_2$  are operators that characterize obtaining residuals  $r_1$  and  $r_2$  from a number  $A_1$ .

$$\begin{aligned} B_3 : A_2 &\rightarrow r_3 \\ B_4 : A_2 &\rightarrow r_4 \end{aligned} \quad (11)$$

where  $B_3, B_4$  are operators that characterize obtaining residuals  $r_3$  and  $r_4$  from a number  $A_2$ . Comparing numbers before and after the shift

$$\begin{aligned} r_1 \equiv r_4 &: P_2 \\ r_2 \equiv r_3 &: P_1 \end{aligned} \quad (12)$$

where  $P_1, P_2$  are the results of comparing the residuals  $r_3$  and  $r_4$ .

$$P : P_1 \vee P_2 \rightarrow p, p \in \{1, 0\}, \quad (13)$$

where  $P$  is an operator that characterizes obtaining the solution  $p$ .

The control scheme for substitution operations is shown in Figure 4.

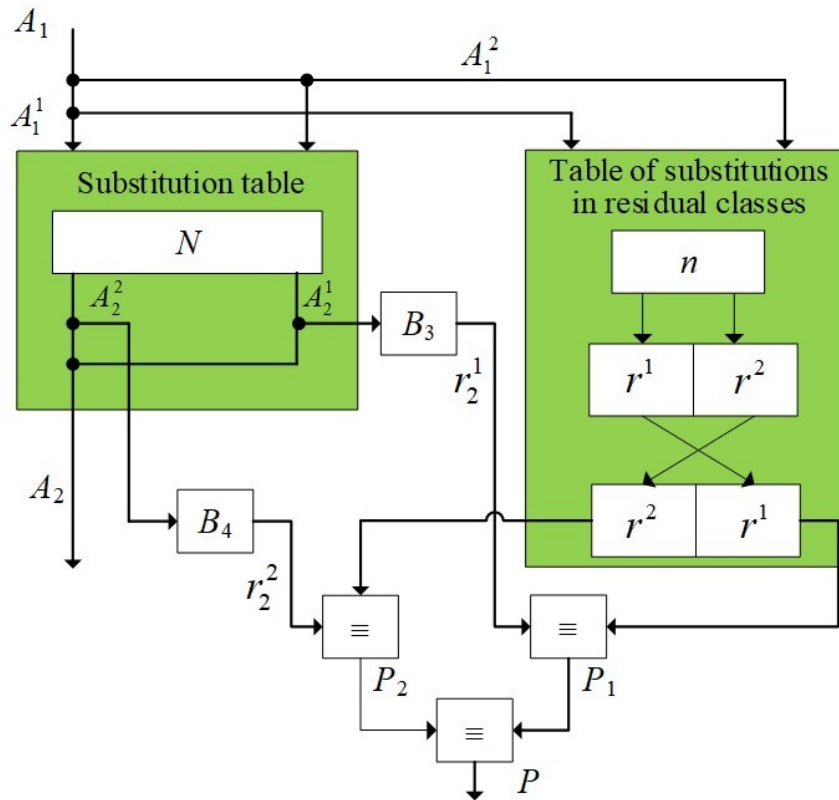


Figure 4 - Control scheme for the substitution operation  
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The functioning of the presented control scheme can be divided into several stages:

The functioning of the presented control scheme can be divided into several stages:

1. At the initial stage, the division of the numbers  $A_1$  and  $A_2$  by the corresponding numbers is carried out, and the definition of the numbers  $A_1 = \langle A_1^1, A_1^2 \rangle$  and  $A_2 = \langle A_2^2, A_2^1 \rangle$ .
  2. From the table of residuals  $n$ , select the residuals  $r_1$  and  $r_2$ .
  3. The remainder ( $r_2^1$  and  $r_2^2$ ) of the numbers are generated after performing the substitution operation and compared with the remainder from the remainder table.
  4. At the last stage, a report is generated about whether the substitution is correct or not ( $P$ ).
- Basic analytical relations [10]:

$$B_3 : A_2^1 \rightarrow r_2^1, \tag{14}$$

where  $B_3$  is an operator that characterizes getting the remainder  $r_2^1$  from a number  $A_2^1$ .

$$B_4 : A_2^2 \rightarrow r_2^2 \tag{15}$$

where  $B_4$  is an operator that characterizes getting the remainder  $r_2^2$  from a number  $A_2^2$ .

Comparing numbers before and after the shift

$$\begin{aligned} r_2^2 &\equiv r^2 : P_2 \\ r_2^1 &\equiv r^1 : P_1 \end{aligned} \tag{16}$$

where  $P_1, P_2$  are the results of comparing the residuals  $r_2^1$  and  $r_2^2$ .

$$P : P_1 \vee P_2 \rightarrow p, p \in \{1, 0\}, \tag{17}$$

where  $P$  is an operator that characterizes obtaining the solution  $p$ .

**Conclusion**

Thus, we have developed control schemes for the main operations of cryptographic transformation, namely addition modulo 2, modulo  $2^{16}$ , shift and substitution. Many additional parameters of the technical diagnostics process have been formed. It is necessary to make adjustments to the functioning model of the encoder, taking into account the process of its technical diagnostics, based on the obtained parameters.

**Конфликт интересов**

Не указан.

**Рецензия**

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**Conflict of Interest**

None declared.

**Review**

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