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МАТЕМАТИЧЕСКАЯ ЛОГИКА, АЛГЕБРА И ТЕОРИЯ ЧИСЕЛ И ДИСКРЕТНАЯ МАТЕМАТИКА/MATHEMATICAL LOGIC, ALGEBRA AND NUMBER THEORY AND DISCRETE MATHEMATICS

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ON FUZZY DISTRIBUTIVE SEMIMODULES

Research article

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Abstract

This paper introduces and studies fuzzy distributive semimodules over commutative semirings. Because fuzzy sums are defined through a supremum, ordinary level cuts need not commute with fuzzy addition unless that supremum is attained; the paper works instead with strict level cuts, which sidestep this attainment problem entirely. Our main result shows that a fuzzy semimodule is distributive precisely when each of its strict level semimodules is distributive. Building on this characterization, we examine how fuzzy distributivity behaves under injective homomorphisms and under isomorphisms. For direct sums, we obtain a preservation result under an explicit decomposition condition on subsemimodules, and we show, without any finiteness assumption, that distributivity of a direct sum forces distributivity of each summand. We further prove that every chained fuzzy semimodule is distributive, and we construct a counterexample showing that the converse fails even when all level semimodules are uniserial. The examples given throughout the paper mark out the boundary between results that transfer directly from module theory and those that call for a genuinely semimodule-specific argument.

Keywords: fuzzy semimodule, distributive semimodule, fuzzy distributive semimodule, chained fuzzy semimodule, level semimodule.

О НЕЧЁТКИХ ДИСТРИБУТИВНЫХ ПОЛУМОДУЛЯХ

Научная статья

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Аннотация

В статье вводится и исследуется класс нечетких дистрибутивных полумодулей над коммутативными полукольцами. В связи с тем, что нечеткие суммы определяются посредством супремума, обычные уровневые сечения не всегда согласуются с операцией нечеткого сложения в случае недостижения супремума. Для устранения данной проблемы используются строгие уровневые сечения. Основной результат устанавливает, что нечеткий полумодуль является дистрибутивным тогда и только тогда, когда каждый его строгий уровневый полумодуль является дистрибутивным. На основе этого результата исследуется сохранение нечеткой дистрибутивности при инъективных гомоморфизмах и изоморфизмах. Для прямых сумм установлено сохранение дистрибутивности при выполнении явного условия разложения подполумодулей, а также доказано, что дистрибутивность прямой суммы влечет дистрибутивность каждого ее слагаемого без дополнительных предположений о конечности. Кроме того, доказано, что всякий цепной нечеткий полумодуль является дистрибутивным, и построен контрпример, показывающий, что обратное утверждение неверно даже в случае унисериальности всех уровневых полумодулей.

Ключевые слова: нечёткий полумодуль, дистрибутивный полумодуль, нечёткий дистрибутивный полумодуль, цепной нечёткий полумодуль, полумодуль уровня.

Introduction

The aim of the present work is to study a fundamental algebraic generalisation of classical modules, namely semimodules, which play an essential role in automata theory, tropical mathematics and theoretical computer science. Distributivity lies at the heart of this structural programme. An R-semimodule M is called distributive if the intersection of A with the sum of B and C is equal to the sum of the intersection of A and B, and the intersection of A and C holds for all subsemimodules A, B and C of M. Classical module theory, and more generally its extension to semimodules, has produced a detailed classification of the structural consequences of distributivity, and a comparable development took place once fuzzy logic was incorporated into algebra [6], [7].

The passage from modules to semimodules is not entirely formal. Semimodules generally lack additive inverses, so several arguments that are routine for modules — those involving homomorphic images, inverse images, or additive decompositions — need extra hypotheses once that inverse structure is gone [6]. Some of the results below are therefore fuzzy analogues that carry over from known distributivity theorems almost unchanged, while others must be reformulated to fit the semimodule setting from the start.

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Level cuts raise a related difficulty. Fuzzy addition is usually defined through a supremum over additive decompositions, and an ordinary, non-strict level cut need not commute with this operation unless the supremum happens to be attained. The use of cut-based representations has also played an important role in recent studies of semiring-valued fuzzy structures [8]. The present paper avoids the issue by working throughout with strict level cuts, which yield a characterization valid without any attainment assumption.

These two observations shape the paper's main contributions. First, we introduce fuzzy distributive semimodules over commutative semirings and characterize them through their strict level semimodules. Second, we determine how fuzzy distributivity behaves under inverse images of injective homomorphisms and under isomorphisms, in each case under hypotheses suited to semimodules rather than modules. Third, we examine direct sums and identify the structural assumptions needed to control their subsemimodules. Recent work on semimodules has likewise shown that direct-sum and direct-summand properties often require additional structural conditions [9], [10].

The rest of the paper is organized as follows. Section 2 recalls the algebraic and fuzzy preliminaries needed later and establishes the strict-level characterization of fuzzy distributive semimodules. Section 3 turns to their behaviour under homomorphisms and isomorphisms. Section 4 takes up direct sums and states the assumptions under which distributivity is preserved. Section 5 relates chained and **level-uniserial** fuzzy semimodules and discusses where the converse implication breaks down. Section 6 collects examples and counterexamples that mark out the scope of the results, and Section 7 closes the paper.

Algebraic and Fuzzy Preliminaries

To ground the algebraic formalism utilized herein, we briefly review semirings, semimodules, and their fuzzy extensions. A semiring is defined as an additively commutative monoid with a zero identity, paired with a multiplicative monoid where multiplication distributes over addition and the additive identity acts as a multiplicative annihilator [1]. A left semimodule extends this structure via scalar multiplication over an additively commutative monoid, satisfying all standard module axioms except the necessity for additive inverses [1], [3]. A subset closed under both internal addition and scalar multiplication constitutes a subsemimodule [3]. We define a subsemimodule as subtractive if the presence of an element and its sum with a second element guarantees the inclusion of that second element [2], [3]. Furthermore, a semimodule is uniserial if its subsemimodules form a totally ordered chain under inclusion, and distributive if the intersection of subsemimodules distributes across their sum. Structure-preserving mappings between these entities, alongside their null spaces and ranges, are termed semimodule homomorphisms [3].

Transitioning to fuzzy topologies, a fuzzy set maps a non-empty set into the real unit interval to assign membership degrees [4]. A fuzzy subset of a semimodule qualifies as a fuzzy semimodule under three conditions: the additive identity holds absolute unity in membership, the membership of any sum is bounded below by the minimum membership of its addends, and scalar multiplication does not strictly decrease an element's membership [5]. A fuzzy subsemimodule independently satisfies these axioms.

Fuzzy Distributive Semimodules

A fuzzy distributive semimodule extends the standard distributive law to fuzzy systems. Let R be a commutative semiring and let M be an R-semimodule. A fuzzy semimodule F on M is called distributive if the identity $A \cap (B + C) = (A \cap B) + (A \cap C)$ holds for any three fuzzy subsemimodules A, B and C of F. This approach replaces rigid crisp bounds by fuzzy subset evaluations, while it still relates to crisp modular structures through strict level cuts.

For a fuzzy subset A of an R-semimodule M and $t \in [0,1)$, write

$$A_t = \{x \in M : A(x) \geq t\}$$

for its strict level cut. When A is a fuzzy subsemimodule, A_t is itself a subsemimodule of M. Strict level cuts are used throughout the paper because they interact naturally with fuzzy addition, which is defined through a supremum, without requiring that the supremum be attained. For fuzzy subsemimodules A and B, intersection and sum are given by

$$(A \cap B)(x) = min\{A(x),B(x)\},$$

$$(A + B)(x) = sup_{x=u+v} min\{A(u),B(v)\}.$$

The next proposition records how these two operations interact with strict level cuts.

3.1. Proposition

Let A and B be fuzzy subsemimodules of an R-semimodule M. Then, for every $t \in [0,1)$,

$$(A \cap B)_t = A_t \cap B_t \text{ and } (A + B)_t = A_t + B_t.$$

Moreover, let F be a fuzzy semimodule on M, and let U be a subsemimodule of F_t*. Define* $\tilde{U}(x) = \{ F(x), x \in U,; 0, x \notin U. \}$

Then $\tilde{U}$ *is a fuzzy subsemimodule of F, and* $(\tilde{U})_t = U$.

Proof. The first equality is immediate from the definition of fuzzy intersection: since $(A \cap B)(x) = min\{A(x),B(x)\}$, the inequality $(A \cap B)(x)_t$ holds exactly when both $A(x)_t$ and B_t. This gives $(A \cap B)_t = A_t \cap B_t$.

For the sum, take $x \in (A + B)_t$, so $(A + B)(x)_t$. By definition,

$$(A + B)(x) = sup_{x=u+v} min\{A(u),B(v)\},$$

so some decomposition $x = u + v$ satisfies $min\{A(u),B(v)\}_t$. Then $u \in A_t$ and $v \in B_t$, and consequently $x \in A_t + B_t$.

Conversely, if $x \in A_t + B_t$, write $x = u + v$ with $u \in A_t$ and $v \in B_t$. Then $min\{A(u),B(v)\}_t$, so $(A + B)(x)_t$ as well. This establishes

$$(A + B)_t = A_t + B_t.$$

For the last claim, let U be a subsemimodule of F_t. If $x,y \in U$, then $x + y \in U$ and

$$\tilde{U}(x + y) = F(x + y) \geq min\{F(x),F(y)\} = min\{\tilde{U}(x),\tilde{U}(y)\};$$

the same inequality holds trivially whenever at least one of x,y lies outside U. Likewise, for $r \in R$ and $x \in U$,

$$\tilde{U}(rx) = F(rx) \geq F(x) = \tilde{U}(x),$$

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and again the inequality is trivial when $x \notin U$, since then $\tilde{U}(x) = 0$. So $\tilde{U}$ is a fuzzy subsemimodule. Because every element of U carries membership above t while every element outside U carries membership zero, it follows that $(\tilde{U})_t = U$.

3.2. Theorem

Let F be a fuzzy distributive semimodule on M. Then, for every $t \in [0,1)$, the strict level semimodule

$$F_t = \{x \in M : F(x) \geq t\}$$

is distributive.

Proof. Fix $t \in [0,1)$ and let U,V,W be arbitrary subsemimodules of F_t. 3.1. Proposition provides fuzzy subsemimodules A,B,C of F with $A_t = U$, $B_t = V$, and $C_t = W$. Since F is fuzzy distributive, $A \cap (B + C) = (A \cap B) + (A \cap C)$; passing to strict level cuts and applying 3.1. Proposition once more turns this into

$$A_t \cap (B_t + C_t) = (A_t \cap B_t) + (A_t \cap C_t),$$

that is, $U \cap (V + W) = (U \cap V) + (U \cap W)$. As U,V,W were arbitrary, F_t is distributive.

3.3. Theorem

Let F be a fuzzy semimodule on M. If F_t is distributive for every $t \in [0,1)$, then F is a fuzzy distributive semimodule.

Proof. Let A,B,C be arbitrary fuzzy subsemimodules of F. Distributivity of F_t gives

$$A_t \cap (B_t + C_t) = (A_t \cap B_t) + (A_t \cap C_t)$$

for every t, and 3.1. Proposition translates this into

$$(A \cap (B + C))_t = ((A \cap B) + (A \cap C))_t, \text{ for every } t \in [0,1).$$

A fuzzy subset is determined by its strict level cuts: if two fuzzy subsets disagreed in membership at some point, any threshold lying between the two values would separate their strict level cuts. Consequently, $A \cap (B + C) = (A \cap B) + (A \cap C)$, and F is fuzzy distributive.

3.4. Corollary

For a fuzzy semimodule F, the following conditions are equivalent:

1. F is a fuzzy distributive semimodule;
2. F_t is distributive for every $t \in [0,1)$.

Proof. This is immediate from Theorems 2.2 and 2.3.

3.5. Proposition

Let F be a fuzzy semimodule. Suppose there exists $t_o \in (0,1)$ such that F_t is distributive for every $t \in [t_o,1)$, and

$$F_t = F_{t_o}$$

for every $0 \leq t < t_o$. Then F is a fuzzy distributive semimodule.

Proof. By hypothesis, F_t is distributive for every $t \geq t_o$. For $t < t_o$we have $F_t = F_{t_o}$, which is itself distributive, so in fact $F_t$is distributive for every $t \in [0,1)$. 3.3. Theorem then gives the conclusion.

Homomorphisms of Fuzzy Distributive Semimodules

This section analyses the algebraic invariance of fuzzy distributive semimodules under homomorphic mappings. Using the structural properties of strict level semimodules, we establish preservation under injective inverse images and under isomorphisms.

4.1. Lemma

Let $f : M \to N$ be an R-semimodule homomorphism, and let G be a fuzzy subsemimodule of N. Then $f^{-1}(G)$ is a fuzzy subsemimodule of M. Moreover, for every $t \in [0,1)$,

$$(f^{-1}(G))_t = f^{-1}(G_t).$$

Proof. For $x,y \in M$,

$$f^{-1}(G)(x + y) = G(f(x + y)) = G(f(x) + f(y)) \geq \min\{G(f(x)),G(f(y))\} = \min\{f^{-1}(G)(x),f^{-1}(G)(y)\}.$$

Similarly, for $r \in R$,

$$f^{-1}(G)(rx) = G(rf(x)) \geq G(f(x)) = f^{-1}(G)(x),$$

so $f^{-1}(G)$ is a fuzzy subsemimodule. Finally, $x \in (f^{-1}(G))_t$ exactly when $G(f(x))_t$, that is, when $f(x) \in G_t$. Hence $(f^{-1}(G))_t = f^{-1}(G_t)$.

4.2. Theorem

Let $f : M \to N$ be an injective R-semimodule homomorphism. If G is a fuzzy distributive semimodule on N, then $f^{-1}(G)$ is a fuzzy distributive semimodule on M.

Proof. By 3.2. Theorem, G_t is distributive for every t. 4.1. Lemma gives $(f^{-1}(G))_t = f^{-1}(G_t)$, and since f is injective, this semimodule is isomorphic to $f(M) \cap G_t$, a subsemimodule of the distributive semimodule G_t. Any subsemimodule of a distributive semimodule is itself distributive, because sums and intersections among its subsemimodules are computed exactly as in the ambient semimodule. So $(f^{-1}(G))_t$ is distributive for every $t \in [0,1)$, and 3.3. Theorem completes the proof.

4.3. Theorem

Let $f : M \to N$ be an isomorphism of R-semimodules, and let F be a fuzzy distributive semimodule on M. Define the fuzzy image f(F) on N by

$$f(F)(y) = F(f^{-1}(y)).$$

Then f(F) is a fuzzy distributive semimodule on N.

Proof. For every $t \in [0,1)$, $(f(F))_t = f(F_t)$. 3.2. Theorem shows F_t is distributive, and since f is an isomorphism, $f(F_t)$ is isomorphic to $F_t$and hence distributive as well. Every strict level semimodule of $f(F)$ is therefore distributive, and 3.3. Theorem shows that $f(F)$ is fuzzy distributive.

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Direct Sums of Fuzzy Distributive Semimodules

This section examines fuzzy distributivity for finite direct sums. Preservation is obtained under an explicit componentwise decomposition condition, while the converse requires no Artinian or Noetherian hypothesis.

5.1. Definition

For fuzzy semimodules F_1 and F_2 on R-semimodules M_1 and M_2, the fuzzy direct sum $F_1 \oplus F_2 : M_1 \oplus M_2 \to [0,1]$ is defined by $(F_1 \oplus F_2)(x_1,x_2) = \min\{F_1(x_1),F_2(x_2)\}$ for all $(x_1,x_2) \in M_1 \oplus M_2$.

The following theorem gives a preservation result for direct sums under a componentwise decomposition assumption.

5.2. Theorem

Let F_1 and F_2 be fuzzy distributive semimodules on M_1 and M_2, respectively. Assume that, for every $t \in [0,1)$, each subsemimodule U of

$$(F_1)_t \oplus (F_2)_t$$

can be represented as $U = U_1 \oplus U_2$, where $U_i \leq (F_i)_t>$ for $i = 1,2$. Then $F_1 \oplus F_2$ is a fuzzy distributive semimodule.

Proof. For every $t \in [0,1)$, $(F_1 \oplus F_2)_t = (F_1)_t \oplus (F_2)_t$. Write $U = U_1 \oplus U_2$, $V = V_1 \oplus V_2$, $W = W_1 \oplus W_2$ for arbitrary subsemimodules of this strict level semimodule. Because F_1 and F_2 are fuzzy distributive, 3.2. Theorem gives

$$U_i \cap (V_i + W_i) = (U_i \cap V_i) + (U_i \cap W_i),\ i = 1,2,$$

and computing componentwise yields $U \cap (V + W) = (U \cap V) + (U \cap W)$. Every strict level semimodule of $F_1 \oplus F_2$ is therefore distributive, and 3.3. Theorem finishes the proof.

5.3. Corollary

Let $F_1,F_2,\ldots,F_n$ be fuzzy distributive semimodules. Suppose that, at each strict level, every subsemimodule of the corresponding finite direct sum decomposes componentwise. Then

$$F_1 \oplus F_2 \oplus \ldots \oplus F_n$$

is a fuzzy distributive semimodule.

Proof. Induction on n, applying 5.2. Theorem at each step, gives the result.

5.4. Theorem

Let $F = F_1 \oplus F_2$ be a fuzzy distributive semimodule. Then F_1 and F_2 are fuzzy distributive semimodules.

Proof. For every $t \in [0,1)$, $F_t = (F_1)_t \oplus (F_2)_t$, and 4.2. Theorem shows this semimodule is distributive. Since $(F_1)_t \oplus \{0\}$ and $\{0\} \oplus (F_2)_t$ are subsemimodules of F_t, they too are distributive, and they are naturally isomorphic to $(F_1)_t$and $(F_2)_t$, respectively. As this holds for every $t \in [0,1)$, 3.3. Theorem shows that F_1 and F_2 are both fuzzy distributive.

Relations with Chained and Uniserial Fuzzy Semimodules

This section elucidates the algebraic interplay between chained and distributive fuzzy semimodules. We establish that chained fuzzy semimodules are distributive and show by counterexample that the converse can fail even when every strict level semimodule is uniserial.

6.1. Definition

A fuzzy semimodule F over an R-semimodule M is classified as chained if its lattice of fuzzy subsemimodules is linearly ordered under pointwise inclusion. Consequently, for any fuzzy subsemimodules A and B of F, the relation $A \leq B$ or $B \leq A$ invariably holds.

6.2. Definition

A fuzzy semimodule F is called level-uniserial if every strict level semimodule

$$F_t = \{x \in M : F(x) \geq t\}$$

is uniserial for every $t \in [0,1)$.

6.3. Theorem

Every chained fuzzy semimodule is a fuzzy distributive semimodule.

Proof. Let F be a chained fuzzy semimodule and let A,B,C be fuzzy subsemimodules of F. Chainedness makes B and C comparable; suppose first that $B \leq C$. Then $B + C = C$, so $A \cap (B + C) = A \cap C$. Since $A \cap B \leq A \cap C$, we also have $(A \cap B) + (A \cap C) = A \cap C$, and the two expressions agree:

$$A \cap (B + C) = (A \cap B) + (A \cap C).$$

The case $C \leq B$ is symmetric. Hence F is fuzzy distributive.

6.4. Remark

The converse of 6.3. Theorem fails: level-wise uniseriality alone does not force a fuzzy semimodule to be chained.

Take $R = \mathbb{N}_0$ and let $M = \mathbb{Z}_4$ carry its natural R-semimodule structure. Its subsemimodules form the chain

$$\{0\} \subset \{0,2\} \subset M,$$

so M is uniserial and, in particular, distributive.

Let $F(x) = 1$ for every $x \in M$. Every strict level semimodule of F then coincides with M, so F is both level-uniserial and fuzzy distributive.

Consider now two fuzzy subsemimodules A and B given by

$$A(0) = 1, A(2) = 0.8, A(1) = A(3) = 0.3,$$
$$B(0) = 1, B(2) = 0.6, B(1) = B(3) = 0.5.$$

Both satisfy the fuzzy subsemimodule conditions, yet

$$A(2) > B(2) \text{ while } A(1) < B(1),$$

so neither $A \leq B$ nor $B \leq A$. Hence F is not chained, even though it is fuzzy distributive and level-uniserial. This example shows that these two properties together still fall short of chainedness.

Examples and Counterexamples

To substantiate the established theoretical framework, this section provides concrete algebraic models that clarify the delineation between fuzzy distributivity and chained structures.

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7.1. Example

Let $R = \mathbb{Q} \geq 0$ denote the semiring of nonnegative rational numbers, and take $M = R$ as an R-semimodule over itself. Define $F(0) = 1$ and

$$F(q) = 1/2$$

for every $q > 0$. For $0 \leq t < 1/2$ this gives $F_t = \mathbb{Q} \geq 0$, while for $1/2 \leq t < 1$ it gives $F_t = \{0\}$. Both level semimodules are distributive, so 3.3. Theorem shows F is fuzzy distributive.

7.2. Example

Let $R = \{0,1\}$ be the Boolean semiring, with $a + b = max\{a,b\}$ and $ab = min\{a,b\}$. Take $M = R$ as a semimodule over itself and define $F(x) = 1$ for every $x \in M$. Since $\{0\}$ and M are the only subsemimodules of M, its subsemimodule lattice is a chain and so distributive. Because $F_t > = M$ for every $t \in [0,1)$, 3.3. Theorem shows that F is a fuzzy distributive semimodule.

7.3. Example

Let $B = \{0,1\}$ be the Boolean semiring and form the product semiring $R = B \times B$. Take $M = R$ as an R-semimodule over itself and define $F(x) = 1$ for every $x \in M$.

The subsemimodules of M are

$$\{0\}, B \times \{0\}, \{0\} \times B, M;$$

their lattice is distributive but not linearly ordered, so M is distributive without being uniserial. As $F_t = M$ for every $t \in [0,1)$, 3.3. Theorem again gives fuzzy distributivity.

The fuzzy subsemimodules corresponding to the two coordinate subsemimodules, however, are incomparable, so F is not chained. This furnishes a second example in which fuzzy distributivity does not force chainedness.

7.4. Example

Let $R = \mathbb{Q} \geq 0$ and set $M_1 = M_2 = R$, with

$$F_1(x) = F_2(x) = 1$$

for every $x \in R$. Each F_i is chained on its own, yet the direct sum $M = M_1 \oplus M_2$ contains the incomparable subsemimodules $R \times \{0\}$ and $\{0\} \times R$. So chainedness need not survive the passage to a direct sum.

Taken together, these examples mark out both the reach and the limits of the results established above. Fuzzy distributivity does not imply chainedness, not even under the level-uniserial condition introduced in Section 5, and the direct-sum examples explain why transferring distributivity from individual components to their direct sum calls for the additional structural assumptions imposed in Section 4.

Conclusion

This paper developed a framework for fuzzy distributive semimodules over commutative semirings. Working with strict level cuts avoids the attainment issue that arises when fuzzy addition is defined through a supremum, and it leads to a precise characterization: a fuzzy semimodule is distributive exactly when each of its strict level semimodules is distributive.

We also reconsidered how this property behaves under semimodule mappings, in a form that respects the absence of additive inverses. In particular, we obtained preservation results for injective inverse images and for isomorphisms without appealing to the unrestricted arguments available for modules. For direct sums, distributivity is preserved under an explicit componentwise decomposition condition, and distributivity of a direct sum implies distributivity of its components without any Artinian or Noetherian assumption.

The relation to chained structures came out clearly as well: every chained fuzzy semimodule is fuzzy distributive, but the converse fails in general, and the counterexamples show that requiring all strict level semimodules to be uniserial still does not recover chainedness at the fuzzy level.

Taken together, these results show that several statements familiar from module theory need extra care once additive inverses are no longer available. The framework developed here may also serve as a starting point for further work on fuzzy automata, weighted algebraic systems, and decision models, though concrete applications in these directions will require problem-specific analysis left for future work. Natural extensions include hesitant fuzzy and interval-valued semimodules, provided their distributive operations and level structures are treated on their own terms.

Конфликт интересов

Не указан.

Рецензия

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Conflict of Interest

None declared.

Review

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